Original construction set for colleges and secondary schools. An innovation.*

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Keywords

Construction sets, constructional toys, elements, forms

1. Introduction

In this short paper we discuss a construction set that seems to be a toy but may have great relevance from the point of view of mathematics and education. Main topics covered: spatial geometry – prisms, a set of convex polygons in constructing solids, future plans.

The author has patent protection on some of his mathematical work, shapes and solids, and some technological elements. As a member of the Club13, he has an opportunity to introduce his inventions – as seen in Figures 1÷4 - in international exhibitions. His goal is to see them one day in schools and institutions. So, the pupils or students could see, how marvellous and inventious solid geometry can be. And what is also important: they could be inspired by the fact, how many things we still don't know.

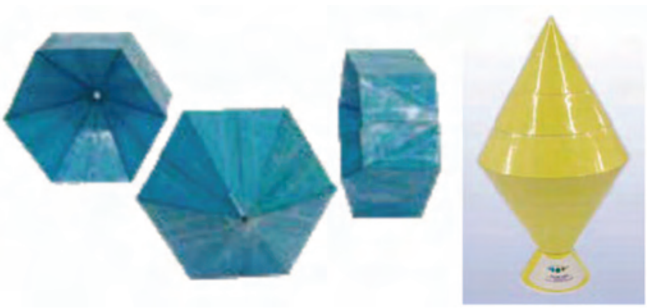

Figure 1. Construction sets.

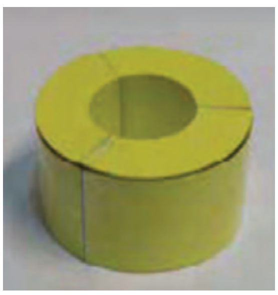

Figure 2. Cones, cylinders, and the ellipse.

2. Descriptions of the construction sets

First construction set: It contains construction parts and elements of the following shape:

Figure 3. Construction set with plane shapes, in the form of a book

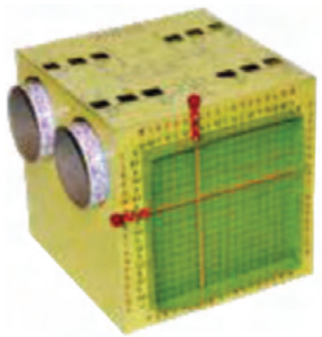

Figure 4. Converting the binomial theorem into a mechanical construction.

1. Example: tetrahedron form construction element, its edges have the following proportions when compared to each other – Figure 5.

Relationship (1):

$a:b:c:d = (3 + 12^{1/2}):(2 + 3^{1/2}):(4 + 12^{1/2}):(20^{1/2} + 15^{1/2})$

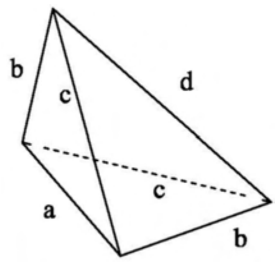

Figure 5. Tetrahedron 1

This causes, that the faces, constituting the solid, have typical angles of 60°, 30° and 90° respectively in triangles of edges a–b–c; and 26.5650°, 63.4350° and 90° respectively in the triangles of edges b–c–d.

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Sudarea şi Încercarea Materialelor

2. Example: The tetrahedron in Example 1, mirrored.

3. Example: tetrahedron form construction element, its edges have the following proportions when compared to each other – Figure 6.

Relationship (2):

$$a:b:c:d:e=(3+12^{1/2}):(2+3^{1/2}):(4+12^{1/2}):(20^{1/2}+15^{1/2}):(8^{1/2}+6^{1/2})$$

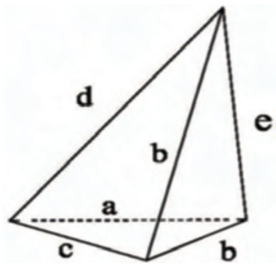


Figure 6. Tetrahedron 2.

This causes, that the faces, constituting the solid, have typical angles of 60°, 30° and 90° respectively in the triangle of edges a–b–c; and 26.5650°, 63.4350° and 90° respectively in the triangle of edges b–c–d, 50.7685°, 90°, 39.2315° respectively in the triangle of edges a–d–e and 45°, 45°, 90° respectively in the triangle of edges b–b–e.

4. Example: the tetrahedron in Example 3, mirrored.

Short description of the first construction set (Figures 7 and 8): The construction set is constituted of 36 elements.

Example 1 +	Example 2+	Example 3+	Example 4	
6 pcs +	6 pcs +	12 pcs +	12 pcs	= 36 pcs of elements.

The elements can be attached with an adhesive tape, so 3 main prisms can be constructed.

Figure 7. The first construction set and its elements.

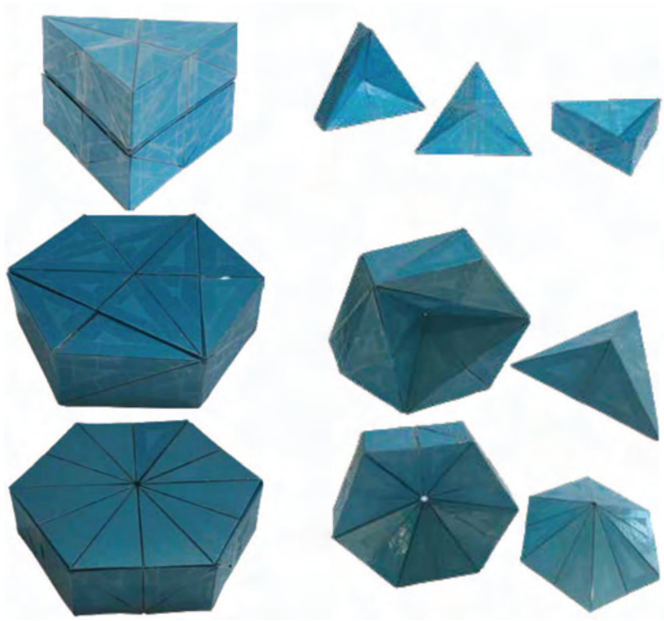


Figure 8. A,B,C constructions.

Second construction set: It contains construction parts and elements of the following shape:

1. Example: tetrahedron (Figure 9) form construction element, its edges have the following proportions when compared to each other:

Relationship (3):

$$a:b:c=(48^{1/2}+27^{1/2}):(192^{1/2}+108^{1/2}):(432^{1/2}+243^{1/2})$$

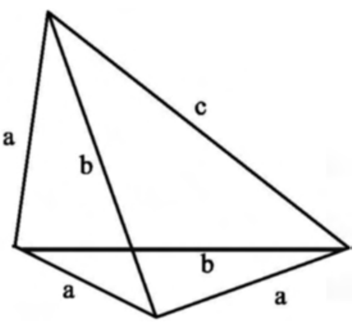


Figure 9. Tetrahedron 3.

This causes, that the faces, constituting the solid, have typical angles of 35.2644°, 54.7355°, 90° respectively in the triangle a–b–c and 45°, 90°, 45° respectively in the triangle a–b–a.

2. Example: The tetrahedron in Example 1, mirrored.

Short description of the second construction set (Figure 10 and 11): The construction set is constituted of 48 elements.

Example 1 +	Example 2	
24 pcs +	24 pcs	= 48 pcs of elements.

These elements can be attached with an adhesive tape, so 3 main prisms can be constructed.

Figure 10. The second construction set.

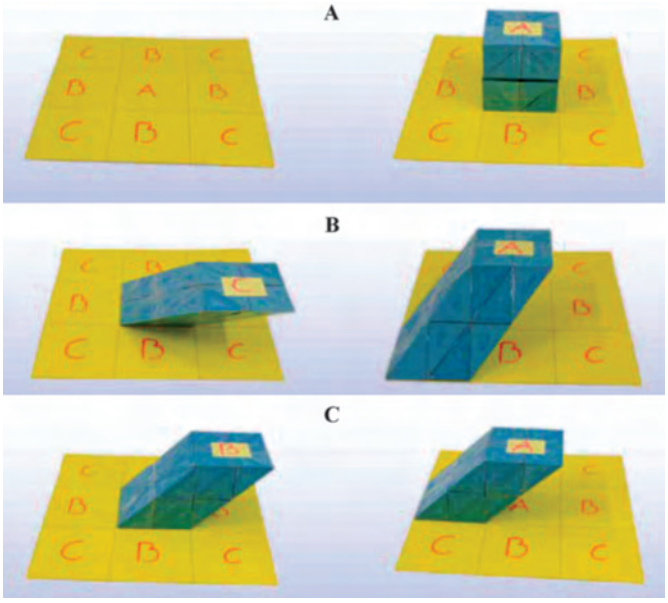


Figure 11. A, B, C, The second construction set.

Third construction set: It contains construction parts and elements of the following shape:

1. Example: tetrahedron (Figure 12.) form construction element, its edges have the following proportions when compared to each other:

Relationship (4):

a:b:c:d=(5.76):(4.18):(7.12):(9.16)

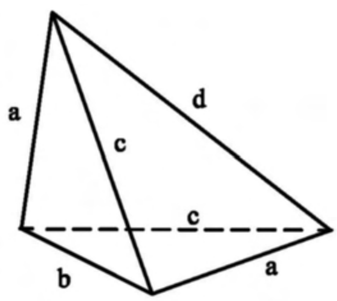


Figure 12. Tetrahedron 4.

This causes, that the faces, constituting the solid, have typical angles of 54°, 36°, 90° respectively in the triangle a–b–c and 38.9735°, 51.0265°, 90° respectively in the triangle a–c–d.

2. Example: The tetrahedron in Example 1, mirrored.

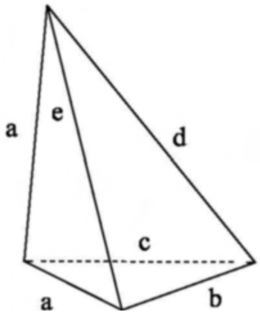


Figure 13. Tetrahedron 5.

3. Example: tetrahedron from construction element, its edges have the following proportions when compared to each other:

Relationship (5):

a:b:c:d:e=(5.76):(4.18):(7.12):(9.16):(8.15)

This causes, that the faces, constituting the solid, have typical angles of 54°, 36° and 90° respectively in the triangle of edges a–b–c; and 38.9735°, 51.0265° and 90° respectively in the triangle of edges a–c–d, 45°, 90°, 45° respectively in the triangle of edges a–e–a and 27.1915°, 90°, 62.8085° respectively in the triangle of edges b–d–e.

4. Example: the tetrahedron in Example 3, mirrored.

Short description of the third construction set (Figure 14.): The construction set is constituted of 60 elements.

Example1 +	Example2+	Example3+	Example4	
10 pcs +	10 pcs +	20 pcs +	20 pcs	= 60 pcs of elements.

The elements can be attached with an adhesive tape, so 1 main prism, and other forms of bodies can be built.

Figure 14. The third construction set

Fourth construction set: It contains construction parts and elements of the following shape: First construction element is a tetrahedron, with faces and edges as follows, in positive direction:

Relationship 6.1

$(3 + 12^{1/2}):(4 + 12^{1/2}):(28^{1/2} + 21^{1/2})$ 90°, 49.1067°, 40.8934°

Relationship 6.2

$(24^{1/2} + 18^{1/2}):(2 + 3^{1/2}):(28^{1/2} + 21^{1/2})$ 90°, 67.7924°, 22.2077°

Relationship 6.3

$(3 + 12^{1/2}):(2 + 3^{1/2}):(4 + 12^{1/2})$ 90°, 30°, 60°

Relationship 6.4

$(3 + 12^{1/2}):(3 + 12^{1/2}):(24^{1/2} + 18^{1/2})$ 90°, 45°, 45°

Second construction element is a tetrahedron, the first construction element mirrored.

Third construction element is a tetrahedron, with faces and edges as follows, in positive direction:

Relationship 7.1

$(3 + 12^{1/2}):(28^{1/2} + 21^{1/2}):(4 + 12^{1/2})$ 90°, 49.1067°, 40.8934°

Relationship 7.2

$(3 + 12^{1/2}):(4 + 12^{1/2}):(2 + 3^{1/2})$ 90°, 30°, 60°

Relationship 7.3

$(2 + 3^{1/2}):(4 + 12^{1/2}):(3 + 12^{1/2})$ 90°, 60°, 30°

Relationship 7.4

$(3 + 12^{1/2}):(28^{1/2} + 21^{1/2}):(4 + 12^{1/2})$ 90°, 49.1067°, 40.8934°

Fourth construction element is a tetrahedron, the 3. construction element mirrored.

Short description of the fourth construction set (Figures 15-18): The construction set is constituted of 72 elements.

Example1 +	Example2+	Example3+	Example4	
12 pcs +	12 pcs +	24 pcs +	24 pcs	= 72 pcs of elements.

The elements can be attached with an adhesive tape, so 4 main prisms can be constructed.

Considering my knowledge and experience, this latter set is the most marvellous. Actually, it is unique of its kind. There is such a huge number of combinations, that a whole life could not be enough to enumerate all the shapes and forms that can be constructed, using 288 elements. Some examples, how complex it is.

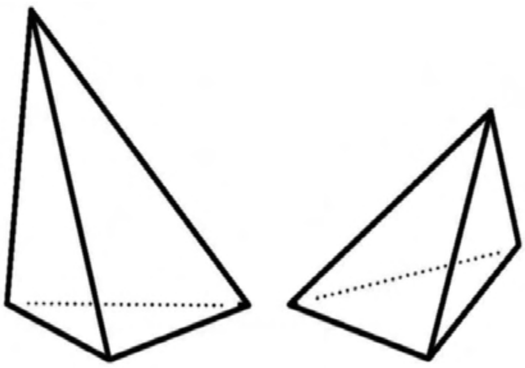


Figure 15. A solid.

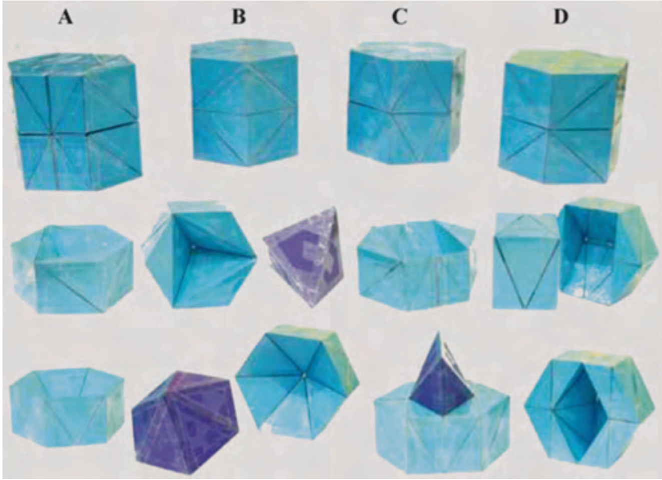


Figure 16. A, B, C, D – The fourth construction set

Figure 17. 144 pcs of tetrahedrons

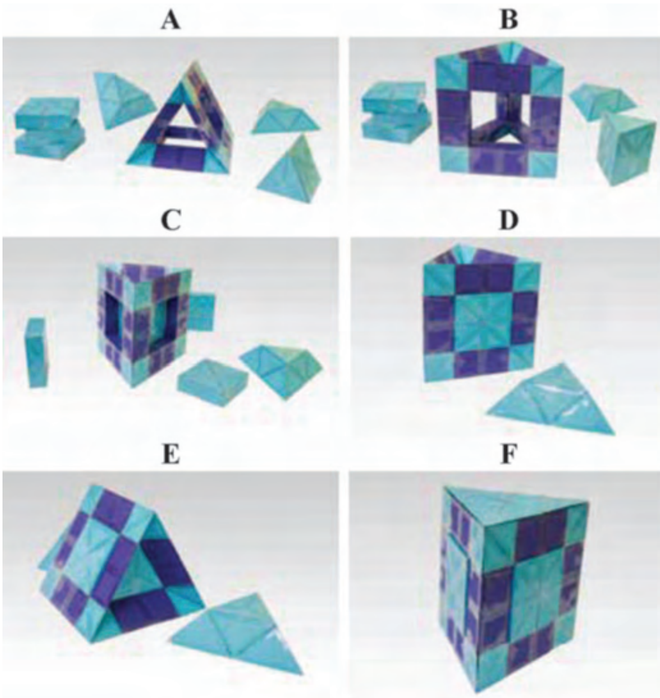


Figure 18. A, B, C, D, E, F -144 pcs of tetrahedrons

The set of the convex polygons (Figure 19). This construction set is a good example how to use a group of convex polygons to create prisms. The key element is the height. You can easily recognise, that two distinct systems can be created. We can call them the cosine system and the sine system (Figures 20÷22). Try to recall the figures! As side a–d keeps growing, the opposite A–D keeps descending in height. In the sine system each and every edge of every prism is equal in length.

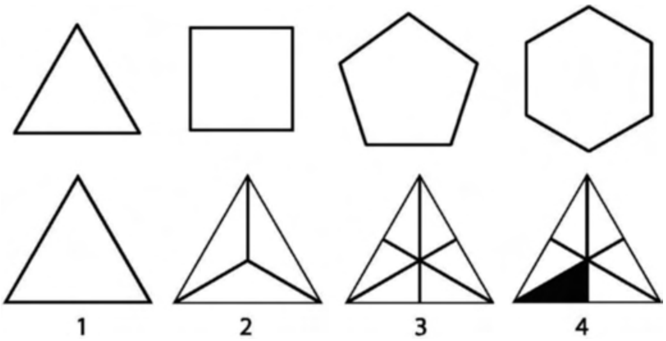


Figure 19. Convex polygons.

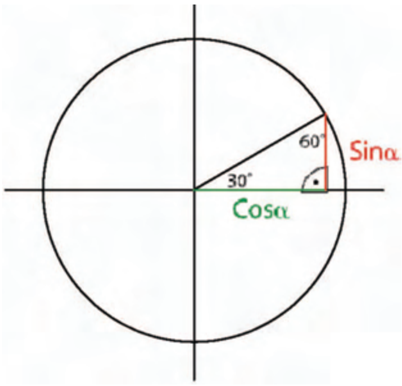


Figure 20. The unit circle.

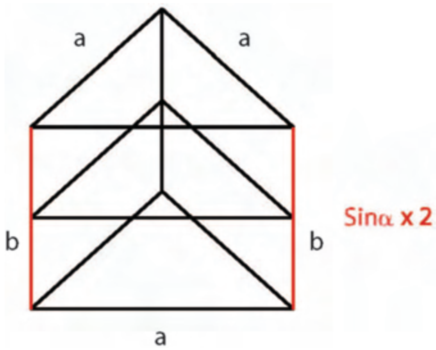


Figure 21. The sine version and the built models.

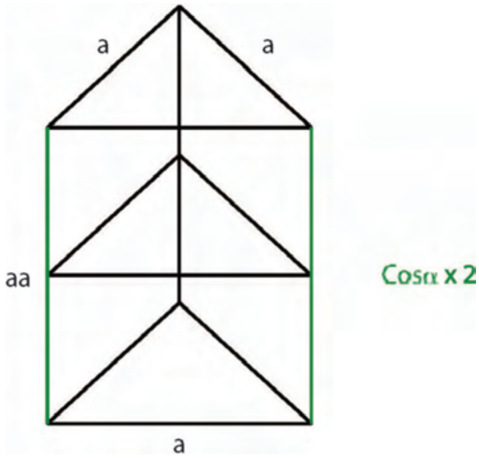


Figure 22. The cosine version, and the built models.

The intersection point. In the table you can see, that regarding any of the construction sets, using 48 elements, you will be able to build a square prism, that will contain an 8-sided pyramid. (Table 1, Figure 23).

The different construction sets

Table 1. The different sets of tetrahedrons.

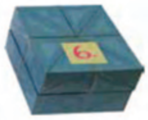
	3	4	5	6
36 pcs tetrahedron				
48 pcs tetrahedron				
60 pcs tetrahedron				
72 pcs tetrahedron				

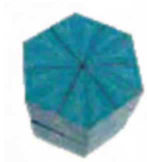
Figure 23. The built models

Explanation to the 1st table. In connection with the 1st table one will ask the question, how many ways are there, to split a square prism (or a cube) into tetrahedrons. The answer is, there are infinite ways. When I first learned about this, I was very surprised. I kept expecting to get nearer to the solution (Table 2. and 3), but this made my life even more difficult. Though, curiosity and the fire inside, are the best motivation. I could not entirely exclude, that there are infinite times infinite distinct systems.

My plans for the future, the atlas. Dear reader. In this short description about this interesting toy, I could only give some basic information. Many questions are not yet answered, even not for me. The size and scope of the subject is huge. To create an overviewing tool for these prisms, tetrahedrons and maybe in the future also conic, cylindrical and various Platonical bodies.

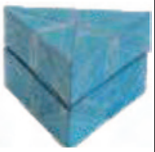
The possibilities of the packages

Table 2. The possibilities of tetrahedrons.

0	1	2	3	4	5	6
0	?	?	triangular prism	square prism	pentagonal prism	hexagonal prism
0	?	?				
0 pcs tetrahedron	12 pcs tetrahedron	24 pcs tetrahedron	36 pcs tetrahedron	48 pcs tetrahedron	60 pcs tetrahedron	72 pcs tetrahedron
0	$A = \sqrt{5292} + 72$	$A = \sqrt{21168} + 144$	$A = \sqrt{47628} + 216$	$A = \sqrt{84672} + 288$	$A = \sqrt{132300} + 360$	$A = \sqrt{190512} + 432$

The different options

Table 3.The different numbers for the different options.

0	?	?					→ etc
pcs of tetrahedron	3	6	9	12	15	18	21
pcs of tetrahedron	6	12	18	24	30	36	42
pcs of tetrahedron	9	18	27	36	45	54	63
pcs of tetrahedron	12	24	36	48	60	72	84
pcs of tetrahedron	15	30	45	60	75	90	105

I am planning to create an atlas, that will contain all my shapes systematically. This is the best way to achieve a clear view. This is going to be the greatest achievement in my life. Today, I can only hope, that I will succeed, and it will be useful for students and mathematicians, and for the rest of the world.

Cones, cylinders, and the ellipse. In the case of cones and cylinders, I wondered if they can be broken up into elements (Figure 24-30). With the help of God, I succeeded. The key element of the solution was a right triangle. It is amazing, that all the way, the solution has been in front of us. That's why I like mathematics, it can bring such turns in my life, like never before. Here new and new systems are standing in front of me, and I have to try to fix them one after the other. The figure shows, how simple the solution was. Let's construct a circle and divide it into three. This sector can be converted into a prism of arbitrary heights. Break it up into 4 elements and multiply them by 6. From these 24 elements, you can construct a wonder. For example, if we use all the 24, we get a simple cylinder, that has cones in it. You can see very well, that what is true in the case of 2D shapes, like convex polygons, is also true for circles. We give them height, we break them into parts, and surely, a variation of construction sets will be created. Can this thing work with ellipses? Really, I don't know. I still have not begun this sort of shapes, because of some technical reasons. But, let's take an example on ellipse. On the figure, we can see the orbit of Earth around the Sun. Let's give it a height in both directions. We can ask the question, what will the following solid be like? Theoretically, it will be one of the figures, and theoretically, there will be multiple constructions possible. It could be considered very interesting, to see beside each other the celestial bodies, from the Sun, to the Pluto. In 2019, I will begin with it, and with the help of God, I will figure out the truth.

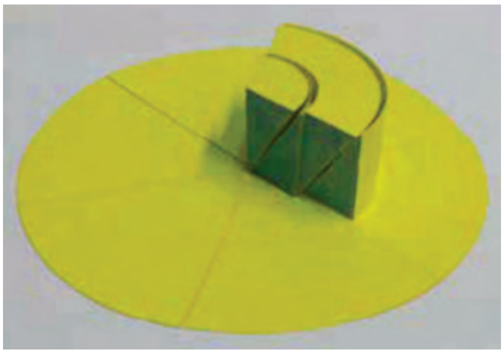


Figure 24. More possibilities.

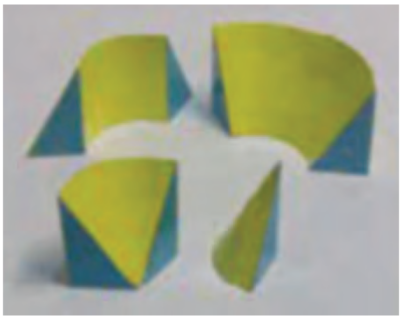


Figure 25. Further plans.

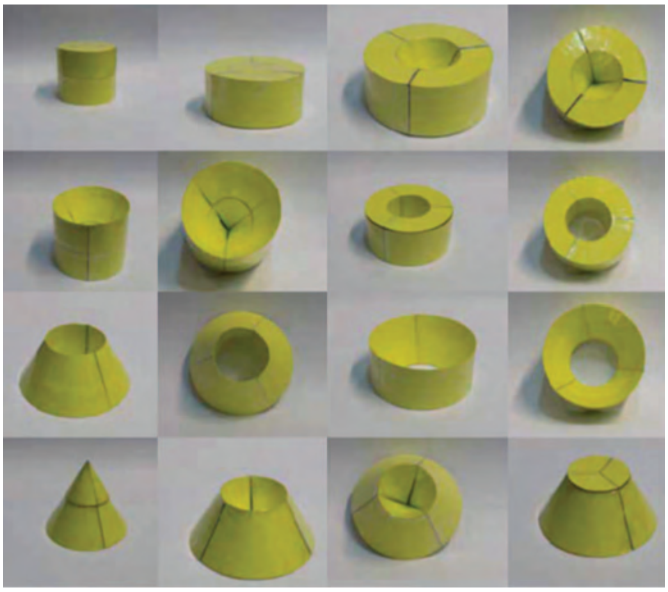


Figure 26. More models.

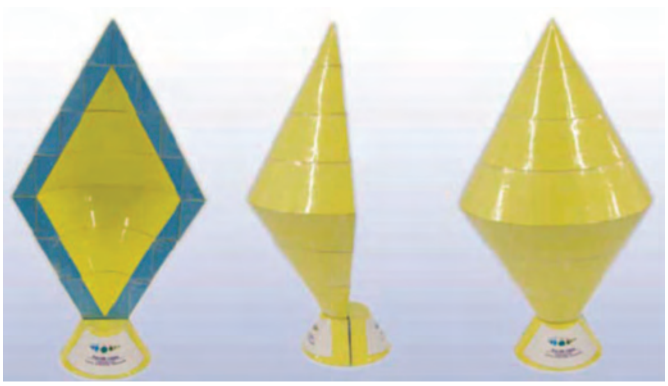


Figure 27. Other options

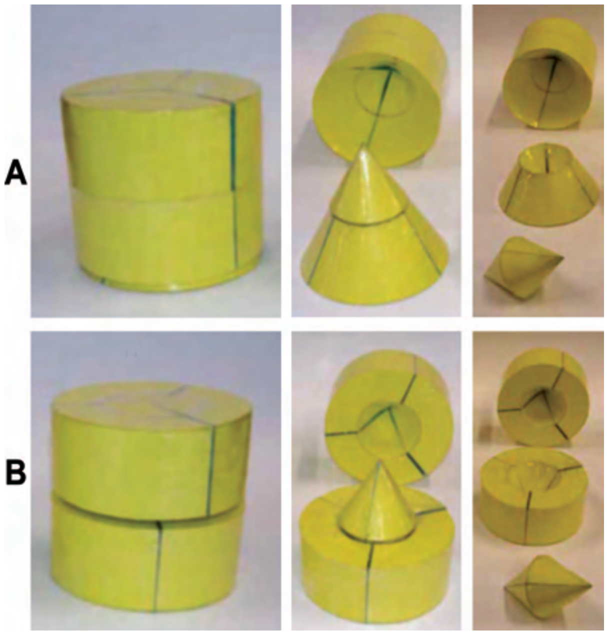


Figure 28. 24 pics of elements.

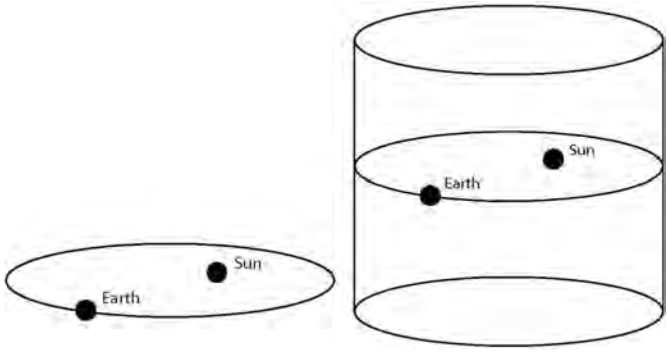


Figure 29. Ellipse.

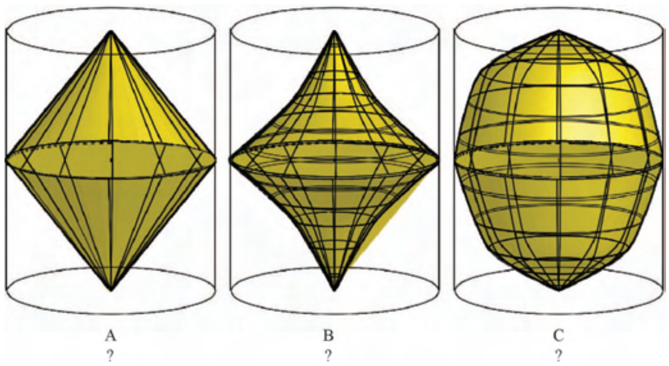


Figure 30. New constructions.

Construction set with plane shapes, in the form of a book

I name every construction set, shape and solid that I construct, after mathematicians. Euclid, as a book, is the plane version of the square based construction set, the plain elements can be converted into 3D solids. Euclid hides three cubes inside. The one that is coloured bronze, can be constructed from 6 squares, using an adhesive tape. The one that is coloured silver, using 6 shapes, can be converted into a pyramid, and using 6 pyramids, a cube can be constructed. I have fit together 8 cross-linked tetrahedrons, so that a four-sided pyramid can be constructed.

In the case of the golden cube, I used the same logic, but it's more difficult. The Euclid-book (Figure 31.) proves, that every construction set can be reduced into plain shapes.

Figure 31. The form of a book, the reduced plain shapes.

Converting the binomial theorem into a mechanical construction (Figures 33A and 33B)

On the figure, we can see a calculation machine. I had constructed it before years. It is a good example of the idea, that if we would like to calculate something, it can be done in several different ways. This machine is set to 9 cm-s. Suppose, that we are in a school. The teacher asks the question: let's calculate $(5+4)^2$. The pupil will do the calculation according to the thesis, and the answer is 81. This machine, as a visual aid to the theorem, will say: I calculate all the combinations $(a+b)^2=81$. In the table, these numbers can be seen on the side of a cylinder. When the teacher asks $(5+4)^3$, we have nothing to do, but to look for the same letters on the side of the cylinder. Now, $(5+4)^{-2}$ will be interesting. Let's take the cylinder out of the machine, and put it in another one, that can read the numbers in a different form, and works with negative indices, too. If the numbers on the cylinder could be rotated, we could calculate $(8+6)^2$. By my personal views, an imaginative cube, with an edge of arbitrary length, and an imaginative square mash, and the numbers of the cubes could be set. 1^2, 2^2, 3^2, 4^2, 5^2, and so on. We push the cube through the mesh, as long as it is, and we receive a number of small cubes. That's how you can calculate all the combinations. This will solve the question, calculate in minutes, set an order and tell all the possible combinations. So, the binomial theorem can be converted into a mechanical machine. It can be mechanized. For the case of powers with the same indices, I have made scratches for the formula $a^2–b^2 = (a+b)(a–b)$. By my calculations (Table 4.), these formulas can also be mechanized. When at home, we take our collection of formulas, and there are so many formulas in it, can all be mechanized? So that every question could be calculated, ordered and all the combinations could be figured out. I don't know.

Table 4. Calculations with the binomial theorem.

$(0 + 9)^2 =$	0	0	18	0	18	18	A	A
$(0.5 + 8.5)^2 =$	1	1	17	1	17	17	B	B
$(1 + 8)^2 =$	2	2	16	2	16	16	C	C
$(1.5 + 7.5)^2 =$	3	3	15	3	15	15	D	D
$(2 + 7)^2 =$	4	4	14	4	14	14	E	E
$(2.5 + 6.5)^2 =$	5	5	13	5	13	13	F	F
$(3 + 6)^2 =$	6	6	12	6	12	12	G	G
$(3.5 + 5.5)^2 =$	7	7	11	7	11	11	H	H
$(4 + 5)^2 =$	8	8	10	8	10	10	I	I
$(4.5 + 4.5)^2 =$	9	9	9	9	9	9	J	J
$(5 + 4)^2 =$	10	10	8	10	8	8	K	K
$(5.5 + 3.5)^2 =$	11	11	7	11	7	7	L	L
$(6 + 3)^2 =$	12	12	6	12	6	6	M	M
$(6.5 + 2.5)^2 =$	13	13	5	13	5	5	N	N
$(7 + 2)^2 =$	14	14	4	14	4	4	O	O
$(7.5 + 1.5)^2 =$	15	15	3	15	3	3	P	P
$(8 + 1)^2 =$	16	16	2	16	2	2	R	R
$(8.5 + 0.5)^2 =$	17	17	1	17	1	1	S	S
$(9 + 0)^2 =$	18	18	0	18	0	0	T	T

$(5+4)^2=25+40+16=81$

$$\begin{array}{c|c|c|c} & 4 & 5 & \\ \hline 4 & 16 & 20 & 4 \\ \hline 5 & 20 & 25 & 5 \\ \hline & 4 & 5 & \end{array} \rightarrow (5+4)^2=25+20+20+16=81$$

$(5+4)^2=25+20+20+16=81$

$$x=2^2 \quad \left(\frac{100}{4}+\frac{80}{4}\right)+\left(\frac{80}{4}+\frac{64}{4}\right)$$

$$\left(\frac{10^2}{2^2}+\frac{10\text{x}8}{2^2}\right)+\left(\frac{10\text{x}8}{2^2}+\frac{8^2}{2^2}\right)$$

$$\boxed{10} \quad \boxed{10\ \ 8} \quad \boxed{10\ \ 8} \quad \boxed{8}$$

$$(5+4)^{-2}=\frac{1}{(5+4)^2}=\frac{1}{81}$$

$$X=2 \quad \left(\frac{\frac{(2\text{x}5)^2}{2^2}}{(5+4)^4}+\frac{\frac{(2\text{x}5)\text{x}(2\text{x}4)}{2^2}}{(5+4)^4}\right)+\left(\frac{\frac{(2\text{x}5)\text{x}(2\text{x}4)}{2^2}}{(5+4)^4}+\frac{\frac{(2\text{x}4)^2}{2^2}}{(5+4)^4}\right)$$

$$\left(\frac{\frac{100}{4}}{6561}+\frac{\frac{10\text{x}8}{4}}{6561}\right)+\left(\frac{\frac{10\text{x}8}{4}}{6561}+\frac{\frac{64}{4}}{6561}\right)$$

$$\left(\frac{\frac{10^2}{2^2}}{81^2}+\frac{\frac{10\text{x}8}{2^2}}{81^2}\right)+\left(\frac{\frac{10\text{x}8}{2^2}}{81^2}+\frac{\frac{8^2}{2^2}}{81^2}\right)$$

$$\left(\frac{1}{262{,}44}+\frac{1}{328{,}05}\right)+\left(\frac{1}{328{,}05}+\frac{1}{410{,}0625}\right)$$

$$\left(\frac{1}{(16{,}2)^2}+\frac{1}{16{,}2\text{x}20{,}25}\right)+\left(\frac{1}{16{,}2\text{x}20{,}25}+\frac{1}{(20{,}25)^2}\right)$$

$$\left(\frac{1}{\left(\frac{162}{10}\right)^2}+\frac{1}{\frac{162}{10}\text{x}\frac{162}{8}}\right)+\left(\frac{1}{\frac{162}{10}\text{x}\frac{162}{8}}+\frac{1}{\left(\frac{162}{8}\right)^2}\right)$$

$$\left(\frac{1}{\left(\frac{(5+4)^2\text{x}2}{10}\right)^2}+\frac{1}{\frac{(5+4)^2\text{x}2}{10}\text{x}\frac{(5+4)^2\text{x}2}{8}}\right)+\left(\frac{1}{\frac{(5+4)^2\text{x}2}{10}\text{x}\frac{(5+4)^2\text{x}2}{8}}+\frac{1}{\left(\frac{(5+4)^2\text{x}2}{8}\right)^2}\right)=\frac{1}{(5+4)^2}$$

$$\boxed{10} \quad \boxed{10\ \ 8} \quad \boxed{10\ \ 8} \quad \boxed{8}$$

Figure 32. Further calculations.

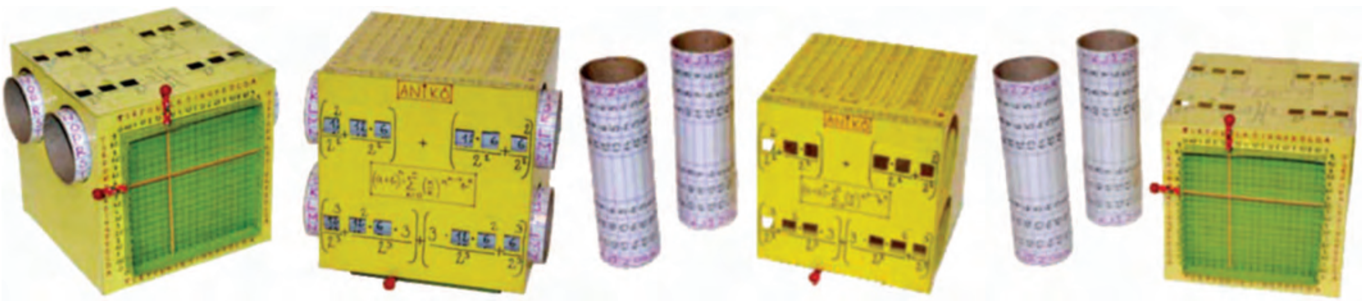


Figure 33. The calculation tool.

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