Resonant ultrasound spectroscopy: theory and some applications

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1. Introduction

All mechanical methods for measuring the elastic properties of laboratory specimens are divided into three types: quasi-static, resonance and time-of flight. These types reflect the ratio of the wavelength of the mechanical signals used to size the specimen under test [1]. The spectrum of modes of some elastic objects contains much information about the object, both at microscopic and macroscopic level. The information that can be derived from such a spectrum includes the entire elasticity matrix as well as information about the object geometry and density. In principle, all of this information can be acquired from single, accurate measurements of an object resonant spectrum. This is accomplished, at least in theory, by solving the inverse problem, which is calculating an object shape and elastic constants based on the knowledge of the experimentally measured resonant spectrum [2].

2. Resonant ultrasound spectroscopy

Resonant ultrasound spectroscopy (RUS) is a means of determining material properties of an elastic object by exciting the resonant frequencies (normal modes) of the object. The spectrum of modes of an elastic object contains much information about the object, both at microscopic and macroscopic level. The information that can be derived from such a spectrum includes the entire elasticity matrix as well as information about the object's geometry and density. In principle, all this information can be acquired from a single, accurate measurement of an object's resonant spectrum [1].

Let Ω be an isolated body, that is, one bounded by a closed stress free surface. Let C_{ijkl} be its elastic stiffness tensor and let ρ be its density; both quantities may vary with position in Ω.

Let ω be a non-negative real number, and a real valued function at position $\overline{r}$ in Ω. Then, the combination $\{\omega,\overline{u}\}$ is a free oscillation or resonance if the real-valued displacement field

$$S(\overline{r},t)=\mathrm{Re}\left(\overline{u}(\overline{r})\exp(j\omega t)\right) \quad (1)$$

where $j=\sqrt{-1}$, satisfies the elastic equation of motion in and the stress-free boundary condition on its surface [2].

The potential energy E_p associated with the displacement field $\overline{u}$ is given by the strain energy

$$E_p=\frac{1}{2}\int_\Omega C_{ijkl}\partial_j u_i \partial_l u_k dV \quad (2)$$

where u_i, i=1,2,3 are the Cartesian components of $\overline{u}$ and we are using Einstein convention for summation. The corresponding kinetic energy E_k is given by E_k = ω^2k; $k=\frac{1}{2}\int_V \rho u_i u_i dV$. The quantity $I=\omega^2 k-E_p$ is stationery if and only if ω and $\overline{u}$ are a resonance of Ω. Let $\{\Phi_\lambda(\overline{r}),\lambda=1,2,...,N\}$ be a set of N specified functions over Ω; $u_i=a_{i,\lambda}\Phi_\lambda$ .We obtain

$$I=\omega^2\alpha\cdot\overline{k}\cdot\alpha-\alpha\cdot\overline{E}\cdot\alpha \quad (3)$$

where α is a vector comprising the juxtaposed components of $a_{i,\lambda}$. I is stationery if and only if

$$\omega^2\overline{k}\alpha=\overline{E}\alpha \quad (4)$$

Eq. (4) is a standard form for the generalized symmetric eigenvalue problem. We shall need to know how perturbations in the sample's material properties, ρ and C, affect its resonance frequencies. We utilize Rayleigh's Principle [5] which tells us how to compute the perturbed eigenvalues of a perturbed hermitian operator without computing the perturbed eigenvectors.

If we perturb the matrices $\overline{k}$ and $\overline{E}$ in (4)

$$\begin{aligned}\overline{k}&\rightarrow\overline{k}+\delta\overline{k}\\ \overline{E}&\rightarrow\overline{E}+\delta\overline{E}\end{aligned} \quad (5)$$

And is possible to represent the perturbation in the squared frequency of the ith resonance by $\omega_i^2\rightarrow\omega_i^2+\delta\omega_i^2$, we can compute $\delta\omega_i^2$ from

$$\delta\omega_i^2=\frac{\alpha^{(i)}\cdot\left(\delta\overline{E}-\omega_i^2\delta\overline{k}\right)\cdot\alpha^{(i)}}{\alpha^{(i)}\cdot\overline{k}\cdot\alpha^{(i)}} \quad (6)$$

where $\alpha^{(i)}$ is the eigenvector associated with ω_i^2. Eq. (6) shows that the modification of the elastic properties, of the density and geometrical shape of a body lead to modification of the position of the resonance maximum, and respectively of the frequency of the eigenmodes of the inspected object.

To visualize the vibration modes as well as for the calculation of RUS frequencies, an adequate numerical code was elaborated in Matlab 2009a.

3. Studied samples

Two types of .prosthesis have been taken into study:

- metallic prosthesis made of Co28.5Cr6Mo, having femoral heads with 51mm diameter (Figure 1a);

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- prosthesis with ceramic heads made of tetragonal zirconium polycrystal stabilized with yttrium oxide (Y-TZP), having a diameter of 28 mm (Figure 1b).

a) b)

Figure1. Studied samples: a) metallic prosthesis; b) ceramic heads

The mechanical properties of the two types of materials have been taken from literature [4], [5].

4. Experimental set-up

The principle scheme of the equipment used for RUS of the femoral head is presented in Figure 2a and a photo of the equipment is presented in Figure 2b. The US transducers are P111 0.06 P3.1 type, being selected so that they should be best damped, to be able to work in wideband.

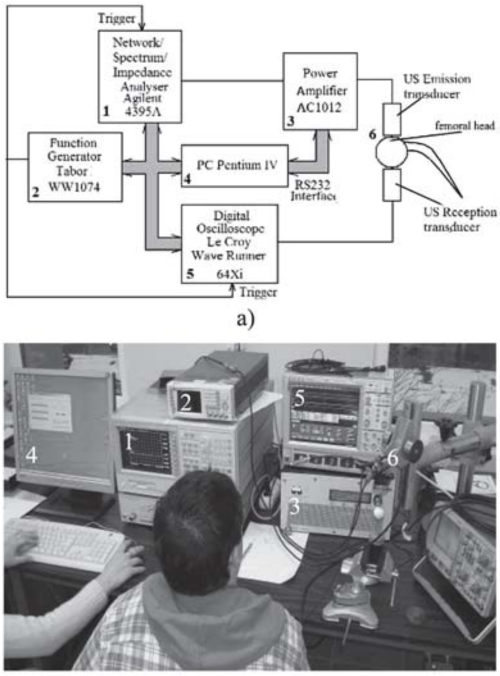

Figure 2. Experimental set-up: a) scheme; b) photo

Agilent 4395A realizes a swept frequency between 50kHz - 2,2 MHz, the signal being applied until 180V amplitude in power amplifier and then applied to US emission transducer. The signal received by the reception transducer is digitized and memorized by the digital oscilloscope Wave Runner 64Xi - Le Croy. The synchronization of the equipment is made with a function generator WW 1074 Tabor that generates the trigger signal.

The command of equipment and the acquisition of the signal are made through a GPIB interface with software developed in Matlab 7.0, less the power amplifier of which command is made by its software through serial RS232 interface. The frequency spectrum was determined using fast Fourier transform (FFT). Due to the used sampling rate, the precision in the determination of frequency is 0.1Hz.

5. Experimental results

The specific problems of quality evaluation of hip prosthesis with ceramic head are the evaluation of density and determination of anomaly as well as the possibility of rapid detection of cracks that can appear during sintering. The correct density of ceramic heads is 6.08 0.01g/cm^3. The head is considered rejected if the density decreases at 6.0 g/cm^3.

In Figure 3 the spectrum of US resonance is presented for the two cases, the defect heads being clearly emphasized. Figure 4 illustrates the US resonance spectrum for ceramic heads with and without cracks due to sintering. Figure 5 shows a ceramic head with a crack emphasized by penetrant liquid method.

The specific problems of quality evaluation of hip prosthesis with metallic head are the anomaly from sphericity, that can

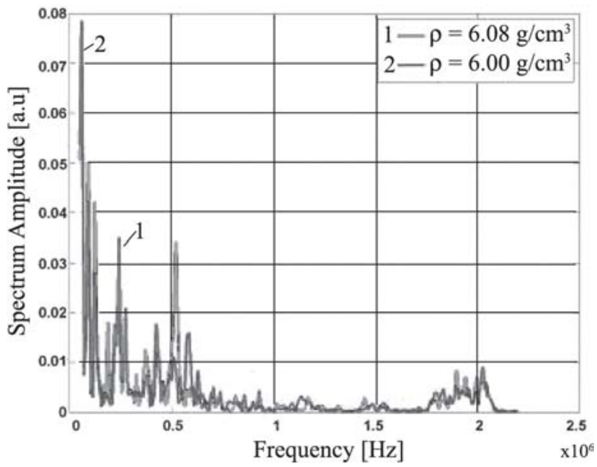

Figure 3. US resonance spectrum for different densities

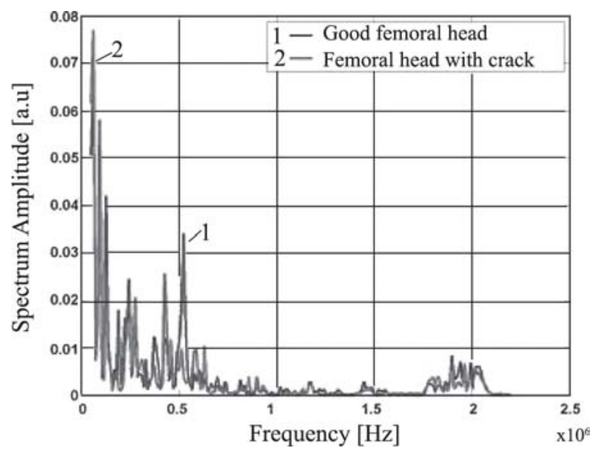

Figure 4. US resonance spectrum for ceramic head without and with cracks

appear due to wearing in the hip articulation, even if the acetabular component is made of high density polyethylene.

A US resonance spectrum both for a new femoral head, correctly dimensioned, and for a head explanted due to deformation is shown in Figure 6. The deformation is clearly visible. The resonance spectrum measured in different conditions presents a relatively high number of peaks, insufficient for obtaining total information about shape, elastic

Figure 5. Ceramic head with crack

constants, density of the controlled prostheses. For this reason, simulators are necessary to establish which peaks are due to geometry and which are due to material properties.

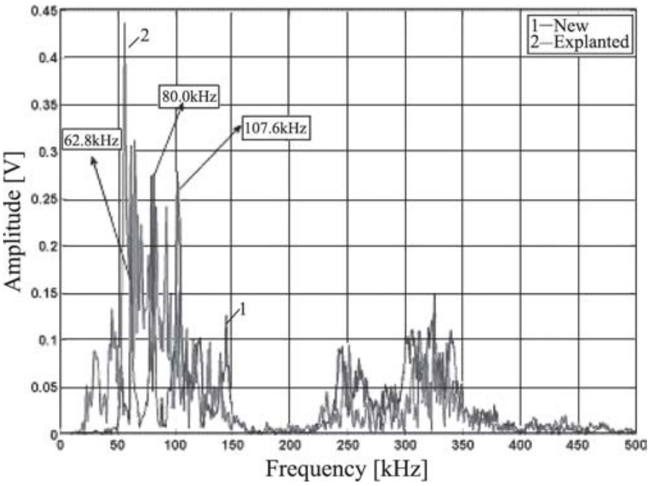

Figure 6. US resonance spectrum for new and explanted heads

In Figures 7, 8 and 9 are presented some of the results obtained through simulation, being chosen those modes that lead to the appearance of maximum deformation of the sphere in the region where the reception transducer is positioned. It can be observed that these peaks correspond to those from the experimentally measured spectrum.

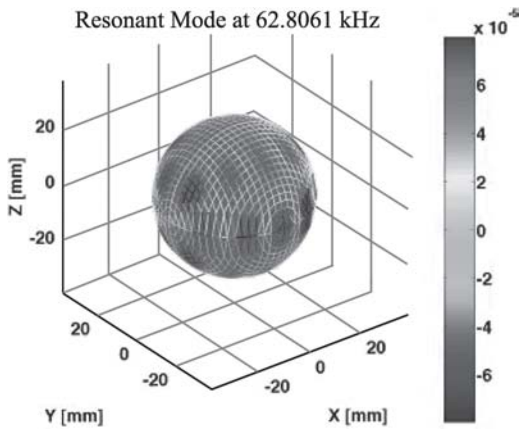

Figure 7. Simulation at 62.8kHz

6. Conclusions

The Resonant Ultrasound Spectroscopy allows, in principle, the determination of elasticity matrix, geometrical dimensions and density. Hypothetically, these parameters can be obtained solving the inverse problem.

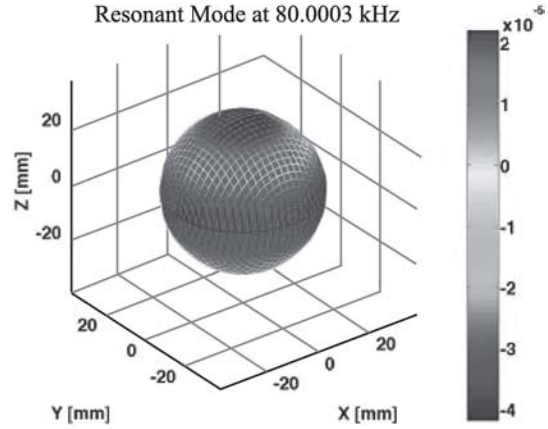

Figure 8. Simulation at 80.0kHz

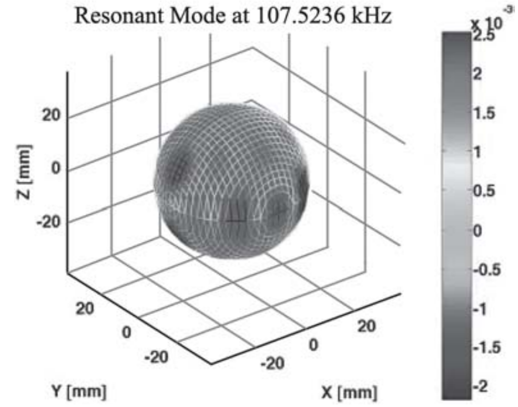

Figure 9. Simulation at 107.5kHz

The number of peaks does not allow to obtain the inverse problem solution, and, in this case, it is necessary to develop a tool for simulation of RUS. This method can be directly applied to nondestructive evaluation of the hip prosthesis made of ceramic and alloys.

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